73-197

AEDC-TR-73-193

cy. 2

TRANSIENT OPTIMIZATION OF A BRAYTON CYCLE REFRIGERATOR

M. J. Triplett
ARO, Inc.

VON KÁRMÁN GAS DYNAMICS FACILITY
ARNOLD ENGINEERING DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
ARNOLD AIR FORCE STATION, TENNESSEE 37389

December 1973

Final Report for Period 1 July 1972 – 30 June 1973

Approved for public release; distribution unlimited.

NOTICES

When U. S. Government drawings specifications, or other data are used for any purpose other than a definitely related Government procurement operation, the Government thereby incurs no responsibility nor any obligation whatsoever, and the fact that the Government may have formulated, furnished, or in any way supplied the said drawings, specifications, or other data, is not to be regarded by implication or otherwise, or in any manner licensing the holder or any other person or corporation, or conveying any rights or permission to manufacture, use, or sell any patented invention that may in any way be related thereto.

Qualified users may obtain copies of this report from the Defense Documentation Center.

References to named commercial products in this report are not to be considered in any sense as an endorsement of the product by the United States Air Force or the Government.

APPROVAL STATEMENT

This technical report has been reviewed and is approved.

William L Simmons

WILLIAM L. SIMMONS
Facility Development Division
Directorate of Civil Engineering

Roland R. Garren

ROLAND R. GARREN
Colonel, USAF
Director of Civil Engineering

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AEDC-TR-73-193	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) TRANSIENT OPTIMIZATION OF A BRAYTON CYCLE REFRIGERATOR		5. TYPE OF REPORT & PERIOD COVERED Final Report--1 July 1972 to 30 June 1973
		6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) M. J. Triplett, ARO, Inc.		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS Arnold Engineering Development Center Arnold Air Force Station, Tennessee 37389		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS Program Element 65802F
11. CONTROLLING OFFICE NAME AND ADDRESS Arnold Engineering Development Center (XON) Arnold Air Force Station, Tennessee 37389		12. REPORT DATE December 1973
		13. NUMBER OF PAGES 30
14. MONITORING AGENCY NAME & ADDRESS(if different from Controlling Office)		15. SECURITY CLASS. (of this report) UNCLASSIFIED
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

Available in DDC.

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

cooling systems
cooling rate
optimization
supercooling
heat transfer

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Brayton cycle refrigerators are frequently used in both plant and flight installations because of their simplicity and acceptable steady-state efficiency. Such installations are normally designed with little thought to the cooldown time required. However, cooldown time can become significant for applications such as batch liquefication or applications where the cooldown time is a large fraction of the total operating time. To test infrared sensors in the AEDC Mark I Aerospace Chamber it will be necessary to cool

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

20, Continued

approximately 35,000 lbs of aluminum to 20°K. With the existing refrigeration system it will take approximately 26 hours to be ready to test. An experimental and analytical study was undertaken to determine ways of reducing this cooldown time. An optimization function inversely proportional to cooldown time was derived which accounted for all known operational and hardware variables. A computer program was written to calculate the optimization function, and analytical results were compared with test results from the existing refrigerator. Results of the study indicated that the inability of the expander to pass full compressor mass flow was the greatest limit to rapid cooldown. An additional expander, used only during cooldown to pass full compressor mass flow, would reduce the cooldown time by 24 percent. Changing only the expander and load to a reheat cycle could reduce the cooldown time by 50 percent.

PREFACE

The work reported herein was conducted by the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC). The work was done by ARO, Inc. (a subsidiary of Sverdrup & Parcel and Associates, Inc.), contract operator of AEDC, AFSC, Arnold Air Force Station, Tennessee. The work was done under ARO Project No. VD227, and the manuscript (ARO Control No. ARO-VKF-TR-73-135) was submitted for publication on October 1, 1973.

CONTENTS

		Page
1.0	INTRODUCTION	
	1.1 Problem Definition	5
	1.2 Solution Approach	5
2.0	ANALYTICAL STUDY	
	2.1 The Optimization Function	6
	2.2 Cycle Analysis	10
	2.3 Results of the Computer Analysis	13
	2.4 The Objective Function	17
	2.5 Optimization	19
3.0	TEST RESULTS	21
4.0	CYCLE VARIATIONS	
	4.1 Description of Considered Variations	23
	4.2 Adding an Expander	23
	4.3 Adding an LN_2/GHe Heat Exchanger	26
	4.4 The Reheat Cycle	26
5.0	CONCLUSIONS	27
	BIBLIOGRAPHY	27

ILLUSTRATIONS

Figure		
1.	Refrigeration System Schematic	6
2.	Temperature-Entropy Diagram	7
3.	Control Volume	8
4.	Load Radiation	10
5.	Refrigerator Capacity	10
6.	Effect of Varying Expander Efficiency	15
7.	Effect of Varying LN_2/GHe Heat Exchanger Effectiveness	15
8.	Effect of Varying GHe/GHe Heat Exchanger Effectiveness	16

Figure		Page
9.	Effect of Varying Pressure Drop After Expander	16
10.	Effect of Varying Pressure Drop Before Expander . . .	16
11.	Effect of Varying Compressor Stage Pressure Ratio . .	16
12.	Effect of Varying T_{stop} and LN_2 Temperature	17
13.	Effect of Varying Expander-On Temperature	17
14.	Typical Load Cooldown	22
15.	Effect of Varying Pressure Drop Before Expander ($\dot{m}$ = 0.35)	24
16.	Effect of Varying Expander Efficiency ($\dot{m}$ = 0.35) . . .	24
17.	Effect of Varying LN_2/GHe Heat Exchanger Effectiveness ($\dot{m}$ = 0.35)	24
18.	Effect of Varying Pressure Drop After Expander ($\dot{m}$ = 0.35)	24
19.	Effect of Varying GHe/GHe Heat Exchanger Effectiveness ($\dot{m}$ = 0.35)	25
20.	Effect of Varying Expander-On Temperature ($\dot{m}$ = 0.35)	25
21.	Effect of Varying Compressor Stage Pressure Ratio ($\dot{m}$ = 0.35)	25
22.	Effect of Varying T_{stop} ($\dot{m}$ = 0.35)	25

TABLES

1.	Comparison Between Analysis and Test	22
2.	Mass Flow Distributions	26

NOMENCLATURE 28

1.0 INTRODUCTION

1.1 PROBLEM DEFINITION

Brayton cycle refrigeration systems are frequently used in plant installations because of component simplicity and acceptable efficiency. Such refrigeration systems normally consist of either a centrifugal or a piston compressor, heat exchangers, and an expander. The most familiar application of the Brayton cycle is the gas turbine. The ideal processes of this cycle are isentropic compression, isobaric heat transfer, and isentropic expansion followed by isobaric heat transfer. The ideal processes of the Brayton cycle refrigerator are usually isothermal compression, isobaric heat transfer, and isentropic expansion. Despite the difference in processes, most literature refers to the refrigeration cycle described either as a modified Brayton cycle or simply as a Brayton cycle.

Refrigerators installed in plants are normally designed for efficient steady-state operation. Brayton cycle refrigerators are also frequently designed for flight use, but they have some problems not dealt with in this study. Refrigerators designed for efficient, steady-state operation and a minimum first cost of installation may not possess optimum cooldown characteristics. When a refrigerator is used to provide the heat sink for a large aerospace testing facility, this additional time may be very expensive. As an example, a typical test run of 12 hrs may require 7 hrs of refrigerator operation to cool the chamber to the required temperature.

An analytical and experimental study was implemented with the goal of minimizing the time required to cool a large panel in a vacuum chamber to 20°K.

1.2 SOLUTION APPROACH

The approach taken in minimizing cooldown time was both analytical and experimental. A preliminary analytical approach was established from first principles. As the cooldown of an existing system was monitored, modifications and additions to the analytical model were made. The study consisted of (1) developing an optimization function, related to cooldown time, which could be used to compare performance, (2) writing a computer program to calculate the optimization function, (3) utilizing the program to perform a sensitivity analysis of operational and hardware variables, (4) running comparative tests with an

operational refrigerator to test operational variables against analytical predictions, and (5) evaluating the effects of possible cycle variations.

2.0 ANALYTICAL STUDY

2.1 THE OPTIMIZATION FUNCTION

A single term was desired to compare the results of a wide variety of modifications to the basic cycle variables. The time required to cool the load from ambient temperature to some arbitrary value in the test region was the obvious choice. However, an exact calculation of time for a particular load can become more involved than is justified for this analysis. Instead, an optimization, inversely proportional to time, was developed. This function was used to compare the relative effects of operational and design parameters for a fixed load mass and material.

Figure 1 is a schematic of the refrigeration system, and Fig. 2 shows a temperature-entropy diagram of the system at a particular operating point. The numbers listed on both figures will be used as subscripts in the equations to be derived. These subscripts will denote particular locations in the thermodynamic cycle (hereafter referred to simply as cycle).

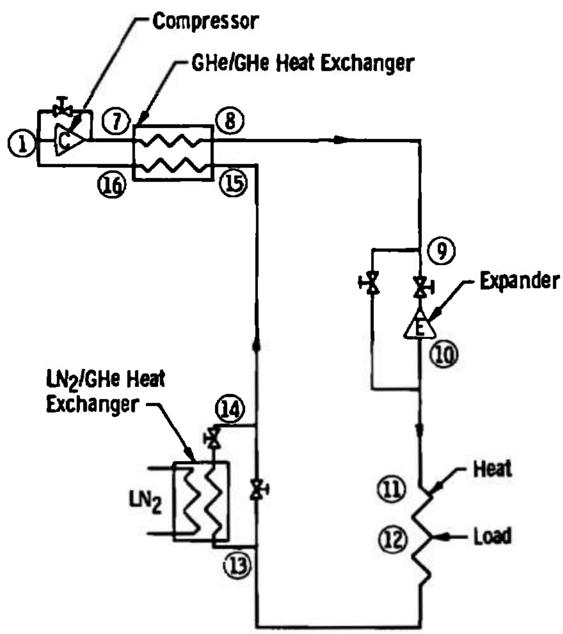

Figure 1. Refrigeration system schematic.

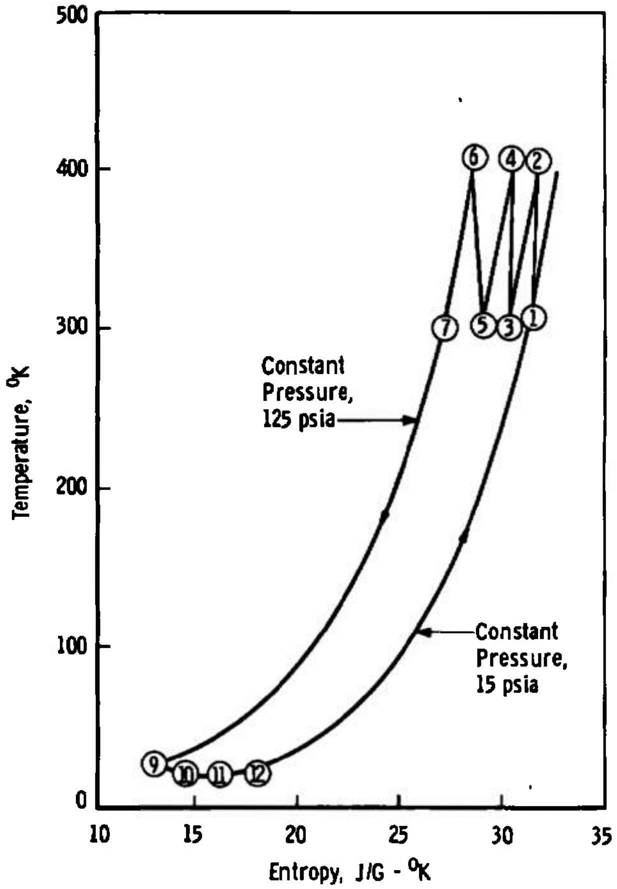

Figure 2. Temperature-entropy diagram.

To derive the optimization function, a control volume of gas was selected inside the load (Fig. 3). The thermal energy added to the gas as it passes through the control volume over an increment of time is expressed as

$$Q = \int_0^t \dot{m}\, Cp(T_{12} - T_{11})dt$$

where

Q = thermal energy
t = time
$\dot{m}$ = mass flow rate
C_p = specific heat at constant pressure
T = temperature

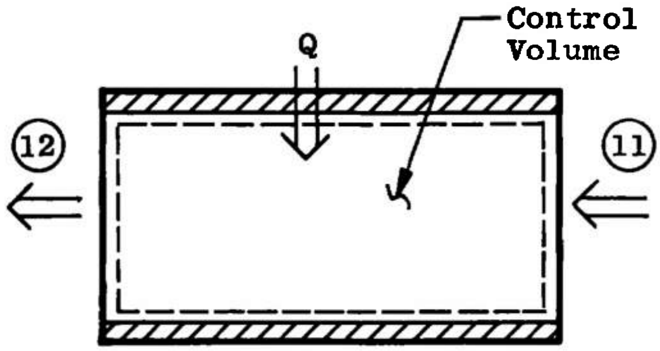

Figure 3. Control volume.

This same amount of thermal energy is taken from the mass of the load (the metal tube through which the gas passes), lowering its temperature as follows:

$$Q = \int_{T_i}^{T_f} WCdT$$

Both the mass flow rate and the gas temperatures T_{11} and T_{12} are unknown functions of time. They are known functions of temperatures and are set by cycle performance, as will be seen later. The specific heat of helium, the flow medium considered in this study, does not vary significantly between 300 and 20°K at the pressures considered (less than 150 psi). For a particular load mass, W, the thermal energy, Q, to be removed can be calculated over any desired temperature increment, since the metal specific heat variation with temperature is known.

If an average mass flow, $\overline{\dot{m}}$, and gas temperature difference $(\overline{T_{12} - T_{11}})$ is used over a short increment of time, the two equations shown previously can be equated and solved as follows:

$$\int_0^t \dot{m}\, Cp(T_{12} - T_{11})dt = \int_{T_i}^{T_f} WCdT$$

$$\overline{\dot{m}}(\overline{T_{12} - T_{11}})Cp \int_{t_1}^{t_2} dt = W \int_{T_i}^{T_f} CdT$$

$$\Delta t = W \int_{T_i}^{T_f} CdT \Big/ \overline{\dot{m}}(\overline{T_{12} - T_{11}})Cp$$

For a given system, the integral $\int_{T_i}^{T_f} C dT$ is a fixed number. Thus, to minimize the time required to cool the system over the increment where $\dot{m}$ and $(T_{12} - T_{11})$ are appropriately constant, it is only necessary to maximize the product of $(\dot{m}) \times (T_{12} - T_{11})$. This product must be taken over many small temperature increments during the cooldown process to be valid, since both $\dot{m}$ and $T_{12} - T_{11}$ vary considerably. It is necessary to form a numerical integral of all these products through the cooldown process to determine a measure of the heat removed by the gas. Cycle variations that maximize this integral also cause the cooldown time to be minimized. The integral

$$\int_{T_{12_i}}^{T_{12_f}} \dot{m}(T_{12} - T_{11}) dT$$

is designated as the optimization function, OPF. To assure accuracy, T_{12} was stepped down in increments of only 2°K. The corresponding value of T_{11} was determined from cycle calculations and is described in Section 2.2 of the present report.

It should be noted that the derivation just described does not consider any external heat leak into the load. It was assumed that all the refrigeration went to cooling the load. Typical heat leaks are calculated for an existing system to justify this assumption. The load consisted of approximately 1200 ft^2 of aluminum panel weighing approximately 4500 pounds. The panel was located inside a chamber with pressures less than 10^{-4} torr. Other panels cooled to 80°K surrounded the load. All panels were painted black (emissivities assumed = 1.0). These surrounding panels are normally at operating temperature when the refrigerator is started. Thus, when a cooldown is started, thermal energy is lost by radiation. As the temperature of the load is cooled below 80°K, the load begins to receive thermal energy by radiation from the surrounding panels. Figure 4 is a plot of the radiation rate of the load as a function of panel temperature. For comparison, a typical refrigeration curve as a function of the temperature of the panel was plotted in Fig. 5. Radiation is seen to be mostly insignificant to the refrigeration available.

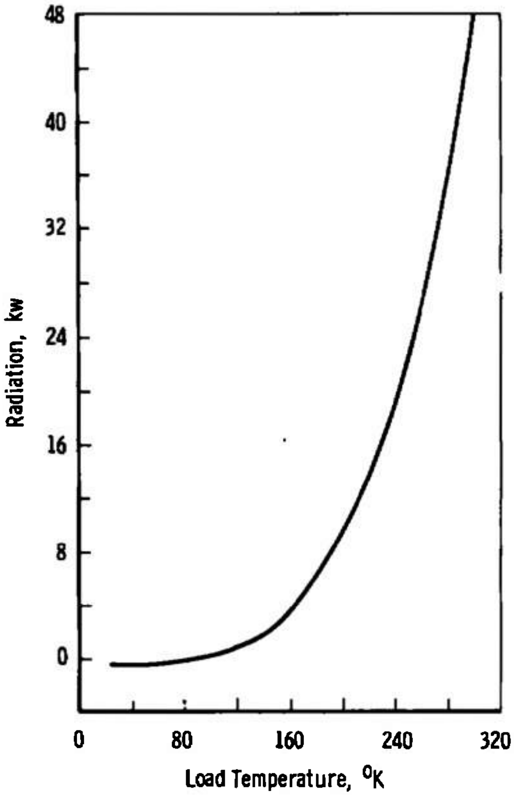

Figure 4. Load radiation.

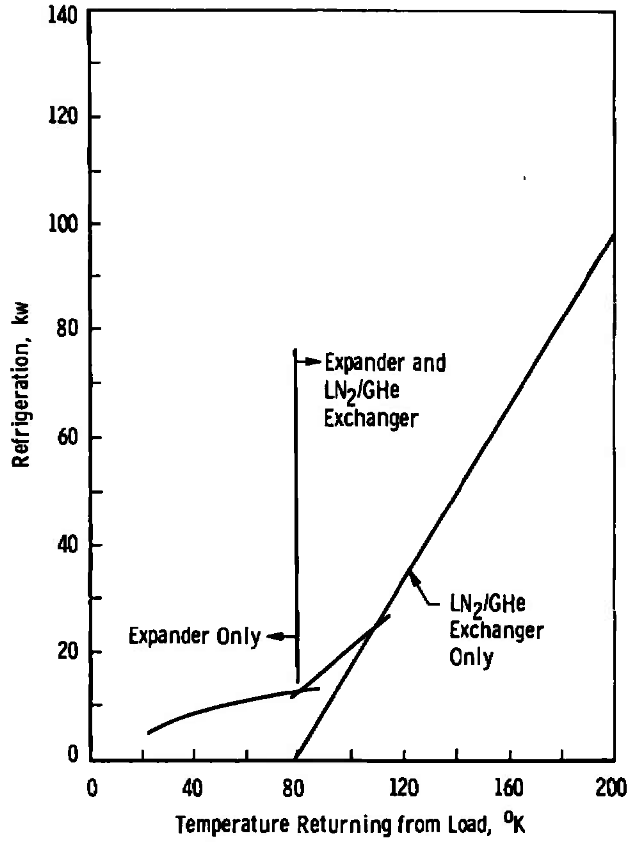

Figure 5. Refrigerator capacity.

Conduction heat transfer into the load can easily be made insignificant by proper design. With a tensile stress of 10,000 psi, a stainless steel rod with a cross section of 0.45 in.^{2} can support the 4500-lb panel. Assuming the rod is only 1 ft long and connects solidly to a 300°K wall, use of the proper thermal conductivity integral shows the heat leak to be only 2.89 w.

2.2 CYCLE ANALYSIS

The value of $T_{12} - T_{11}$ is set by the performance capability of the cycle components. In the refrigeration system studied in this analysis the compression process was carried out with a three-stage piston compressor with water intercooling. Each stage was equipped with water-jacketed cylinder walls and was represented in the analysis as a polytropic process. Heat exchange was essentially an isobaric process, and the expander was corrected from the ideal isentropic process by an expander efficiency.

Equations for the various parameters were derived, using T_{12} as an input variable to ultimately calculate T_{11} and thus $T_{12} - T_{11}$. A value for $\dot{m}$ was input as a function of T_{12}.

With the expander bypassed, only the liquid nitrogen/gaseous helium (LN_2/GHe) heat exchanger provides cooling. A parameter describing the performance of such a heat exchanger is the heat exchanger effectiveness. This parameter is defined as the actual heat transferred divided by the maximum possible heat transfer. For the LN_2/GHe heat exchanger the effectiveness is

$$\epsilon_{LN_2} = \frac{T_{13} - T_{14}}{T_{13} - T_{LN_2}}$$

Provisions were made to input heat leaks as well as pressure drops into the cycle. Heat leaks were entered as temperature changes for convenient insertion into the cycle calculations. The modified effectiveness equation for the LN_2/GHe heat exchanger, considering heat leak, is

$$\epsilon_{LN_2} = (T_{13} - T_{14} + \Delta T_{13\text{-}14})/(T_{13} - T_{LN_2})$$

Similarly, the effectiveness of the GHe/GHe heat exchanger, with heat leak, is

$$\epsilon_{GHe} = (T_{16} - T_{15} + \Delta T_{15\text{-}16})/(T_7 - T_{15})$$

Using the above equations, the sequence of calculations for T_{11} with the expander bypassed is

$$T_{13} = T_{12} + \Delta T_{12\text{-}13}$$

$$T_{14} = T_{13}(1 - \epsilon_{LN_2}) + T_{LN_2}\,\epsilon_{LN_2} + \Delta T_{13\text{-}14}$$

$$T_{15} = T_{14} + T_{14\text{-}15}$$

$$T_8 = \epsilon_{GHe} T_{15} + T_7(1 - \epsilon_{GHe}) + \Delta T_{7\text{-}8}$$

$$T_7 = 300^{\circ}\text{K by water cooling}$$

$$T_{16} = (1 - \epsilon_{GHe})T_{15} + T_7\,\epsilon_{GHe} - \Delta T_{15\text{-}16}$$

$$T_9 = T_8 + \Delta T_{8\text{-}9}$$

$$T_{10} = T_9 + \Delta T_{9\text{-}10}$$

$$T_{11} = T_{10} + \Delta T_{10\text{-}11}$$

When the expander is operating and the LN_2/GHe heat exchanger is being utilized, all temperatures except T_{10} are calculated just as before. The temperature leaving the expander, T_{10}, can be calculated from the pressure ratio across the expander and the expander efficiency, as follows (note that k = Specific heat ratio):

$$T_{10} = T_9\left\{1 - \epsilon_{exp}\left[1 - (P_9/P_{10})^{\frac{1-k}{k}}\right]\right\} + \Delta T_{9-10}$$

where

$$P_9 = P_7 - (\Delta P_{7-8} + \Delta P_{8-9})$$

$$P_7 = P_1(P_2/P_1)^3$$

$$P_{10} = P_1 + \Delta P_{16-1} + \Delta P_{15-16} + \Delta P_{14-15} + \Delta P_{13-14} + \Delta P_{12-13} + \Delta P_{11-12} + \Delta P_{10-11}$$

When the LN_2/GHe heat exchanger was not being utilized,

$$T_{14} = T_{13} + \Delta T_{13-14}$$

A computer program was written for performing the calculations listed above on a digital computer using a FORTRAN IV language. Input data were the system parameters such as stage pressure ratio, P2P1, heat leaks, ΔT, pressure drops, ΔP, and various efficiencies. Also, codes were established for utilizing the proper equations depending on the particular configuration. This was necessary to account for operational variables such as T_{exp}, the temperature of the gas coming from the load (T_{12}) at which the expander is added to the cycle (°K), and T_{stop}, the temperature of the gas coming from the load (T_{12}) at which the LN_2/GHe heat exchanger is removed from the cycle (°K). It was assumed in this study that the LN_2/GHe heat exchanger would always be in operation at the start of a cooldown. The variable T_{stop} was used so that the computer would use the proper equations after the LN_2/GHe exchanger was removed from the cycle. It is always removed from the cycle at some point because it begins to add thermal energy to the refrigerant rather than removing it. The expander could be brought on at any temperature, T_{exp}, including ambient, but it was assumed that it would remain operational for the rest of the cooldown.

During the cooldown process T_{12} was reduced 2°K at a time. The mass flow was a program input as a function of T_{12}. For each value of

T_{12} a value of T_{11} was calculated utilizing the appropriate cycle equations. The product $\dot{m}(T_{12} - T_{11})$ was then formed and stored as a function of T_{12}. When T_{12} was reduced from 300 to 20°K, these values of $\dot{m}(T_{12} - T_{11})$ were taken from storage, and the optimization function,

$$OPF = \int_{T_{12_i}}^{T_{12_f}} \dot{m}(T_{12} - T_{11})dT$$

was calculated numerically, using the trapezoidal rule.

2.3 RESULTS OF THE COMPUTER ANALYSIS

To obtain some basic knowledge about the system, it was desirable first to determine the parameters that had the most significant effects on cooldown time. The more important parameters could then be studied in more detail with the goal of improving performance.

Examination of the cycle equations revealed nine parameters per study, as follows: (1) compressor stage pressure ratio; (2) LN_2/GHe heat exchanger effectiveness; (3) GHe/GHe heat exchanger effectiveness; (4) expander efficiency; (5) pressure drop upstream of the expander; (6) pressure drop downstream of the expander; (7) the temperature at which the expander is turned on; (8) the temperature at which the LN_2/GHe heat exchanger is turned off; and (9) the temperature of the LN_2. Since an operational refrigerator was available for comparison with the analytical model, reasonable starting values for each parameter were available. These basic values were as follows:

1. P2P1 = 2.04
2. $\epsilon_{LN_2} = 0.98$
3. $\epsilon_{GHe} = 0.988$
4. $\epsilon_{exp} = 0.73$
5. $\Delta P_{11-12} = 2.0$ psi
6. $\Delta P_{7-8} = 2.0$ psi
7. $T_{exp} = 100°K$
8. $T_{stop} = 100°K$
9. $T_{LN_2} = 80°K$

A systematic series of computer calculations was performed in which each parameter was varied above and below the basic value while all other parameters were held constant at the basic value. During this first set of runs it was assumed that the piping in the system would cause sufficient pressure losses to cause a variation in mass flow during cooldown. It was known from previous observations of the operating system that full mass flow was possible through the load only at temperatures below approximately 20°K. An equation was derived for determining the mass flow at higher temperatures utilizing the Fanning Equation. This did not give a wide variation with mass flow, and the resulting computer studies indicated that the expander should be turned on at 300°K to minimize cooldown time. Early experimental runs showed the derived mass flow equation to be incorrect, and the cooldown was actually slowed by bringing the expander on at 300°K. Study of mass flow measurements to the chamber were not meaningful from a quantitative standpoint because of unresolved instrument error. However, it did become apparent that the flow was choked somewhere in the expander. An expression was derived which could be used to calculate the mass flow through the expander as a function of cycle conditions. The equation for mass flow into the expander is

$$\dot{m} = \rho A V$$

where ρ is the gas density, A is the flow area, and V is the flow velocity. Utilizing the equation of state and the equation for the velocity of sound, the equation becomes

$$\dot{m} = \frac{p}{RT} \times A \times \sqrt{kgRT}$$

where R is the gas constant, k is the specific heat ratio, and g is the gravitational constant. Pressures and areas did not change when the expander was brought on line. The only variable in the above equation is T or T_9, the temperature of the gas entering the expander. Thus, the form of the mass flow equation is

$$\dot{m} = \frac{K}{\sqrt{T_9}}$$

with the constant K evaluated to be 1.565 for $\dot{m}$ = 0.35 lbm/sec and T_9 = 20°K.

Utilizing this mass flow variation, each parameter was again systematically varied above and below the nominal value. Results of these

calculations are shown in Figs. 6 through 13. Five of the parameters, expander efficiency, LN_2/GHe heat exchanger effectiveness, GHe/GHe heat exchanger effectiveness, pressure drop after the expander, and pressure drop before the expander, are seen to be quite linear. Compressor stage pressure ratio is a quadratic function but only varies 0.04 percent over the reasonably assumed range of operation. A straight-line curve fit of OPF versus P2P1 was made using least squares and the computer output. As might be expected from closer examination of the cycle equations, the effects on the optimization function of LN_2 temperature variation and the temperature at which the LN_2/GHe heat exchanger is turned off are coupled. The variation in optimization function with varying expander-on temperatures is significantly nonlinear but is conveniently represented by a quadratic function.

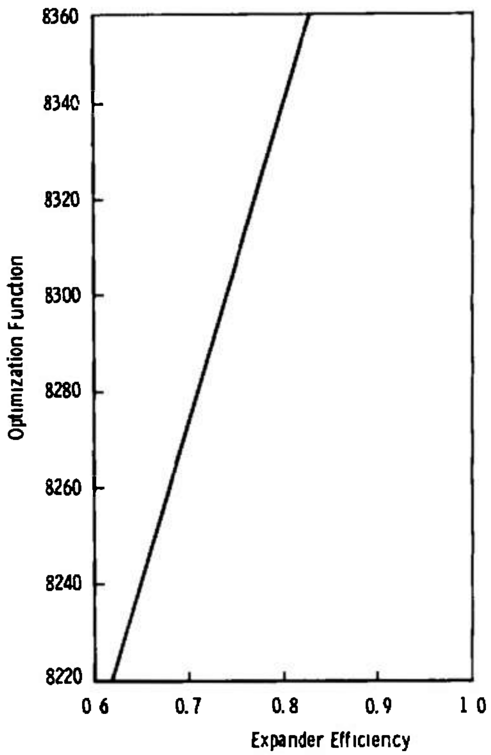

Figure 6. Effect of varying expander efficiency.

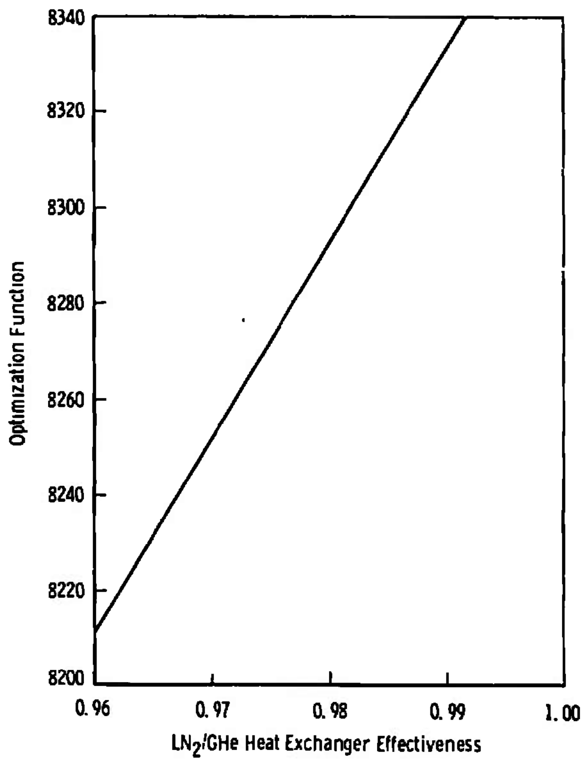

Figure 7. Effect of varying LN_2/GHe heat exchanger effectiveness.

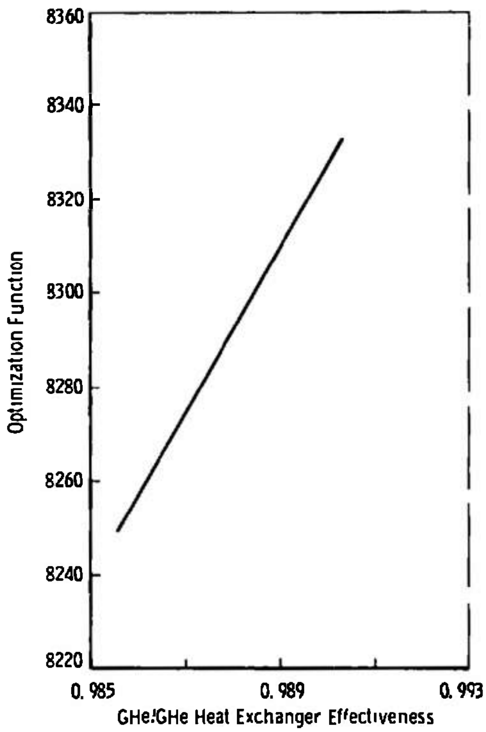

Figure 8. Effect of varying GHe/GHe heat exchanger effectiveness.

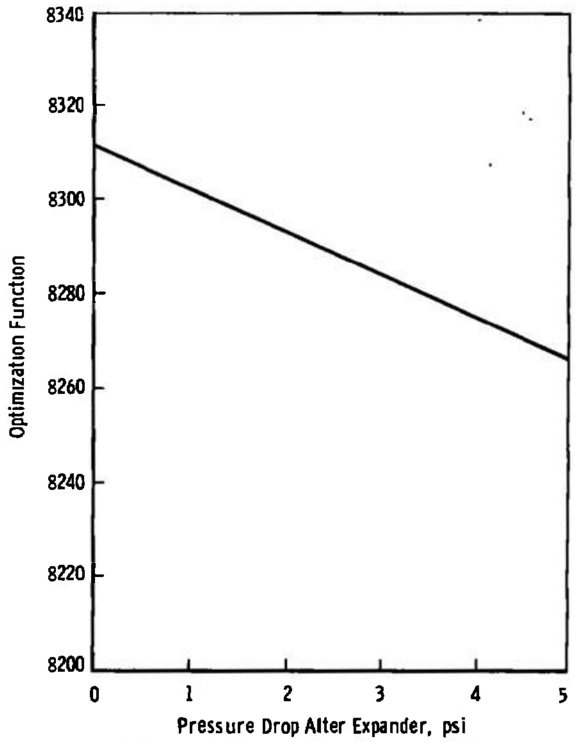

Figure 9. Effect of varying pressure drop after expander.

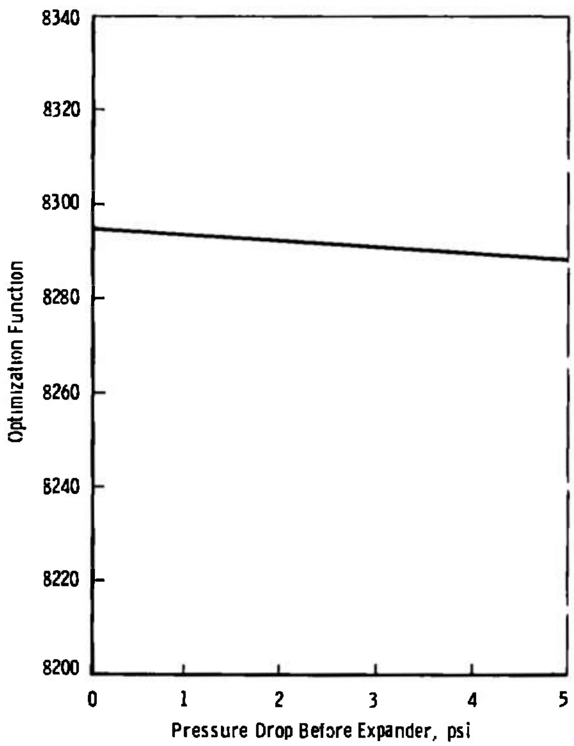

Figure 10. Effect of varying pressure drop before expander.

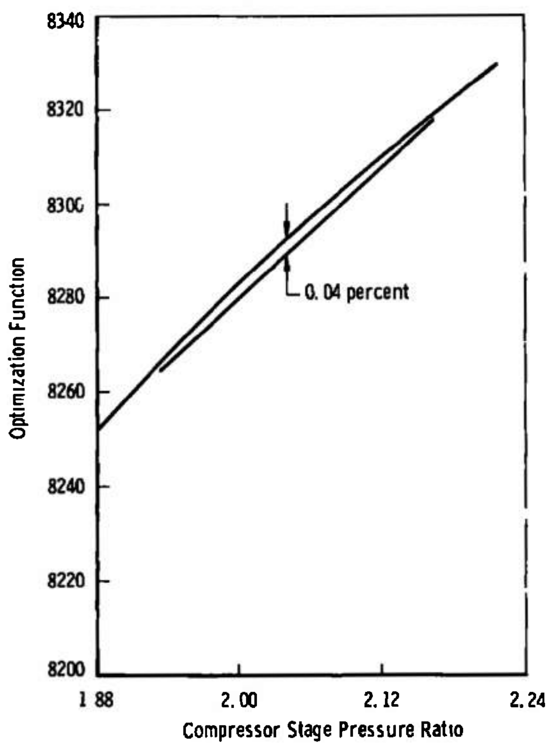

Figure 11. Effect of varying compressor stage pressure ratio.

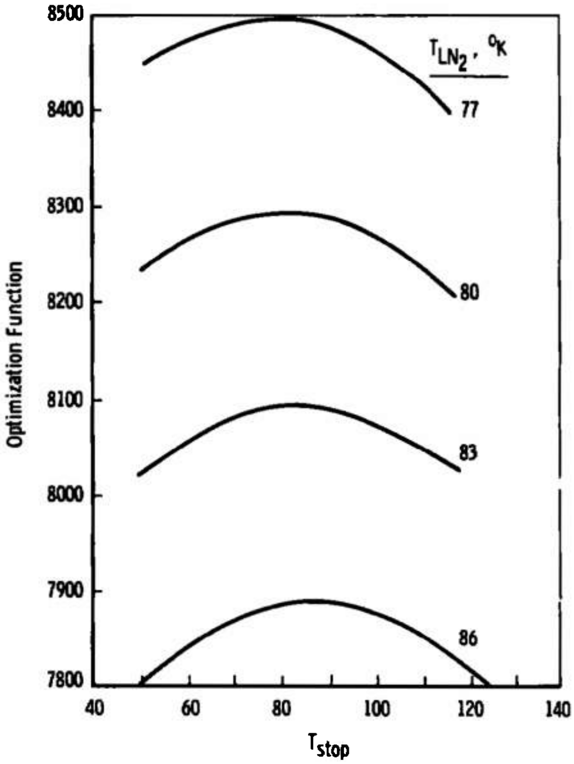

Figure 12. Effect of varying T_{stop} and LN_2 temperature.

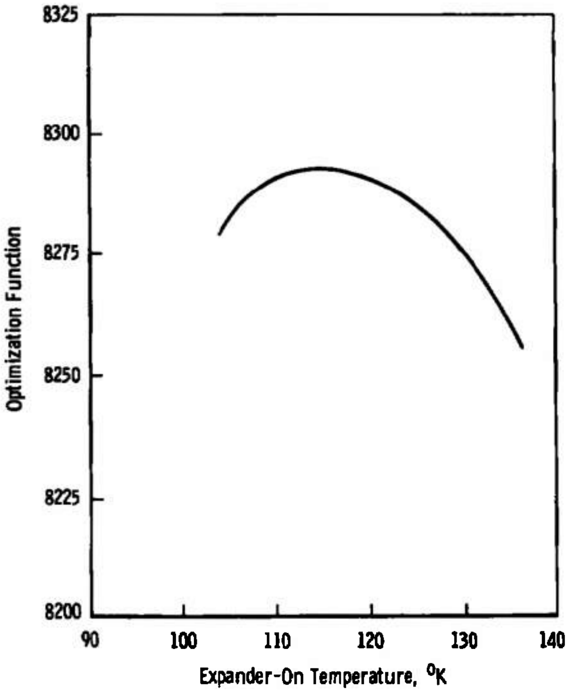

Figure 13. Effect of varying expander-on temperature.

2.4 THE OBJECTIVE FUNCTION

To study the effects each variable has on cooldown time, it is desirable to have a common basis of comparison. To utilize the methodology developed in the field of operations research for optimization of the system's performance, a single equation describing the effects of each variable is required. This single equation that includes all the variables is called an objective function. In this study the objective function becomes an equation that is used to calculate the optimization function as a function of the hardware and operational variables. The same symbol, OPF, was used for the objective function. In the following explanation, x_i for i = 1 to n is used as a substitute for the variables ϵ_{LN_2}, etc., for brevity.

For any selected set of hardware and operational variables the objective function is

$$OPF = OPF(Basic) + d(OPF)$$

where d(OPF) can be written

$$d(OPF) = \frac{\partial(OPF)}{\partial x_1} dx_1 + \frac{\partial(OPF)}{\partial x_2} dx_2 + \cdot \cdot \cdot + \frac{\partial(OPF)}{\partial x_n} dx_n$$

$\frac{\partial(OPF)}{\partial x_i}$ is recognized as the slope of the plots of OPF versus variable i found in Figs. 6 through 13. The amount a particular variable is changed from Basic is dx_i. The amount the objective function is changed from Basic because of all variable changes is d(OPF).

With the variables set at their basic values (Section 2.2), OPF(Basic) is 8292.3001.

As an example, two variables will be used to demonstrate how the objective function was derived.

$$OPF = OPF(Basic) + d(OPF)$$

$$d(OPF) = \frac{\partial OPF}{\partial \epsilon_{exp}} d\epsilon_{exp} + \frac{\partial OPF}{\partial \epsilon_{LN_2}} d\epsilon_{LN_2}$$

$$d(OPF) = 649.2(\epsilon_{exp} - 0.73) + 8101(\epsilon_{LN_2} - 0.98)$$

$$OPF = 8292.3001 + 649.2\ \epsilon_{exp} + 8101\ \epsilon_{LN_2} - 8412$$

$$OPF = 649.2\ \epsilon_{exp} + 8101\ \epsilon_{LN_2} - 120.6$$

which agrees within 0.2 percent of the values calculated by the computer program for $\epsilon_{exp} = 0.80$ and $\epsilon_{LN_2} = 0.99$.

The technique described above was used with linear variations of OPF versus x_i. For nonlinear variations such as OPF versus T_{exp}, a different curve-fitting technique was used. Figure 13 was represented by the following equation, holding all other variables constant:

$$OPF = 8269 - 0.036\ T_{exp}^2$$

As an example, this equation will be combined with the equation previously derived for varying ϵ_{exp} and ϵ_{LN_2}.

$$OPF = 649.2\ \epsilon_{exp} + 8101\ \epsilon_{LN_2} - 0.036\ T_{exp}^2 + k$$

where k is a constant evaluated by substituting basic values of OPF, ϵ_{exp}, ϵ_{LN_2}, and T_{exp}. By using these techniques, the objective equation was finally determined to be as follows:

$$\begin{aligned} OPF = {} & 649.2\ \epsilon_{exp} + 8101\ \epsilon_{LN_2} + 17,700\ \epsilon_{GHe} \\ & - 9.02\ \Delta P_{11-12} - 1.28\ \Delta P_{7-8} \\ & + 231.8\ P2P1 - 0.069\ T_{stop}^2 + 11.02\ T_{stop} \\ & - 68\ T_{LN_2} - 0.036\ T_{exp}^2 - 12,585 \end{aligned}$$

2.5 OPTIMIZATION

With an objective function, the second step in utilizing operations research methodology to obtain an optimum solution is to select the appropriate constraints. In this study the maximum stage pressure (P2P1) is 2.04 and is set by hardware limitations. Values for heat exchanger effectiveness (both ϵ_{GHe} and ϵ_{LN_2}) and the expander efficiency (ϵ_{exp}) must be less than 1.0 for thermodynamic reasons. All pressure drops obviously must be greater than zero. With the refrigeration process starting at ambient conditions, the expander must be turned on and the LN_2/GHe heat exchanger turned off at some temperature less than 300°K. The liquid nitrogen temperature under normal conditions will be greater than 77°K since that is its boiling temperature at ambient pressure. However, the system used in the experimental study was pressurized, and 80°K was used as the basic temperature. With the reasons given above, the constraints are listed below:

$$P2P1 \leq 2.04$$

$$\epsilon_{GHe} \leq 1.0$$

$$\epsilon_{LN_2} \leq 1.0$$

$$\epsilon_{exp} \leq 1.0$$

$$\Delta P_{7-8} \geq 0$$

$$T_{exp} \leq 300°K$$

$$T_{stop} \leq 300°K$$

$$T_{LN_2} \leq 80°K$$

Some of the above conditions could have been predicted without such optimization procedures. However, intuition does not tell the amount of influence each variable has on the objective function when conditions vary from optimum. Determining these quantities is referred to as a sensitivity analysis. The variation in the objective function with a change in variable i is $\partial(OPF)/\partial x_i$. For each variable considered, the sensitivity value is listed below.

$$\frac{\partial(OPF)}{\partial(P2P1)} = 231.8$$

$$\frac{\partial(OPF)}{\partial(\epsilon_{GHe})} = 17,700$$

$$\frac{\partial(OPF)}{\partial(\epsilon_{LN_2})} = 8101$$

$$\frac{\partial(OPF)}{\partial(\epsilon_{exp})} = 649.2$$

$$\frac{\partial(OPF)}{\partial(\Delta P_{7-8})} = -1.28$$

$$\frac{\partial(OPF)}{\partial(\Delta P_{11-12})} = -9.02$$

$$\frac{\partial(OPF)}{\partial(T_{exp})} = -0.072\ T_{exp}$$

$$\frac{\partial(OPF)}{\partial(T_{stop})} = -0.069\ T_{stop} + 11.02$$

$$\frac{\partial(OPF)}{\partial(T_{LN_2})} = -68$$

The relative effects of these variables can be expressed in more meaningful terms by recalling that OPF was derived to be inversely proportional to time. For the system that was tested, a typical cool-down took approximately 7 hrs with OPF = 8292 with $\epsilon_{GHe} = 0.988$. Using the sensitivity coefficient, the predicted change in OPF with a drop of ϵ_{GHe} to 0.987 would be

$$\Delta OPF = 17,700(0.988 - 0.987) = 17.7$$

The predicted cooldown time would be

$$\frac{8292-17.7}{8292} \times 7 \text{ hrs} = 6.98 \text{ hrs}$$

A corresponding change of 0.01 in ϵ_{exp}, in time, is

$$\Delta OPF = 649.2(0.98 - 0.97) = 6.49$$

$$\text{time} = \frac{8292 - 6.49}{8292} \times 7 = 6.99 \text{ hrs.}$$

3.0 TEST RESULTS

During the conduction of a series of tests in the Mark I Aerospace Chamber at AEDC, cooldown data were taken for comparison with analytical studies. The Mark I Chamber is an 84-ft-high by 42-ft-diam vacuum vessel entirely lined with panels which are cooled to 77°K. Pressures are approximately 10^{-6} torr during testing. A 100-ft-long by 6-ft-wide aluminum panel is suspended inside the liner and cooled to less than 20°K with gaseous helium. Refrigeration is supplied with a Brayton cycle refrigerator, and helium from the refrigerator flows through passages in the aluminum panel. Cycle variables are approximately those listed previously as Basic.

Platinum temperature sensors were attached to the panel on the inlet and exhaust manifolds. During the cooldown different procedures were used to check assumptions made during analysis. A typical plot of cooldown data is shown in Fig. 14. It should be noted that it is extremely difficult to obtain what might be called laboratory-type data on such a large system while it is undergoing tests for some other purpose. There was a separate circuit which was also cooled, but it took only a small amount of refrigerant. It was turned on at various unknown times during the cooldown process. Heat leaks were found during some cooldowns and were eliminated. One heat leak was actually located by referring to computer printouts of predicted temperatures and checking against some installed temperature indicators. No indicators were installed at the heat leak point. These problems did not prevent the cooldown from being completed as required, but it did make comparisons with analytical work less accurate.

Cooldown began with the 2500-lb aluminum GHe/GHe heat exchangers at various temperatures. These heat exchangers are very well insulated, and if only a few days pass between cooldowns, they

stay cold. The usual operational procedure is to run the refrigerator in a bypass mode around the chamber to cool the entire refrigerator, including the heat exchangers, prior to starting cooldown on the load. However, this is not always possible. In the computer program which calculates the optimization function, the warm heat exchanger is simulated as a heat leak.

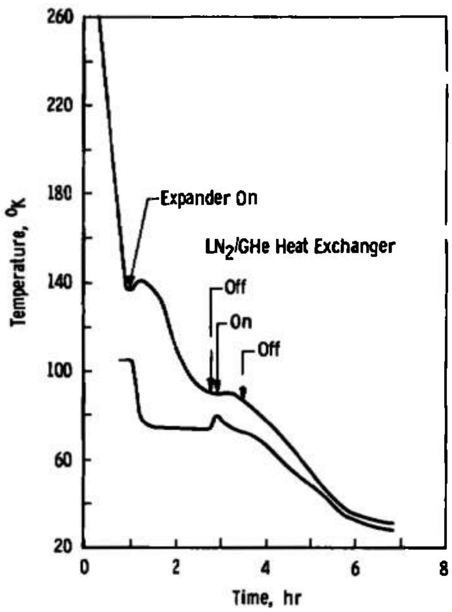

Figure 14. Typical load cooldown.

Table I is a tabulation of the operating conditions and cooldown times observed during tests. Only data for cooldowns which ran smoothly are shown. Measured temperatures of 73 and 78°K for T_{stop} are probably from 5 to 7°K lower than the actual gas temperature but were used as the point to take the LN_2/GHe heat exchanger off the line.

Table 1. Comparison Between Analysis and Test

Conditions			Test	Analysis	
T_{exp}, °K	T_{stop}, °K	Initial Heat Exchanger Temp, °K	Percent Change in Time for Basic = 6.9 Hr	OPF	Percent Change in OPF from Basic
220	90	169	11.9	7018	18
122	107	183	2.98	8181	1.35
130	73	179	1.49	8156	1.67
125	78	178	2.8	8205	1.06

Values of OPF were calculated for the particular operating conditions using Eq. (1), from Section 2.4.

The results presented confirm the predicted effects of changes in the operational variables T_{exp} and T_{stop}. Starting the expander at a high temperature slows the cooldown time because of insufficient mass flow. Varying T_{stop} over the range from 107 to 73°K only varies the overall cooldown time a small amount.

4.0 CYCLE VARIATIONS

4.1 DESCRIPTION OF CONSIDERED VARIATIONS

There are many possible cycle variations that might be considered if this were a study of a proposed system rather than an existing one. As it is, only changes that either overcame serious existing problems or could be added with a minimum of hardware modification are considered.

Both experimental test and analysis confirmed the limiting component on the test refrigerator to be the expander and its inability to pass full compressor mass flow at higher temperatures. The results of a study of the system assuming full compressor mass flow throughout the cooldown process are given in Section 4.2. This could be accomplished by adding an expander in parallel with the existing one. After the system was cooled to such a temperature that one expander could handle full mass flow, one expander could be shut down.

Some literature on cryogenic refrigerators assumes an additional LN_2/GHe heat exchanger located in between two GHe/GHe heat exchangers. Such an addition could be provided for the existing system and is considered in Section 4.3.

Reheat cycles are frequently used to improve efficiency. This modification is checked for transient performance in Section 4.4.

4.2 ADDING AN EXPANDER

Since adding an expander would overcome serious difficulties in the existing system, a complete analysis was run in that configuration. An objective function was derived using methods described previously, assuming full compressor mass flow during the entire cooldown. The computer output is plotted in Figs. 15 through 22. The resulting equation for OPF is as follows:

$$OPF = 1488P2P1 - 8.141\ \Delta P_{7-8} - 58.55\ \Delta P_{11-12} + 13,179\ \epsilon_{GHe} + 4949\ \epsilon_{LN_2} + 4157\ \epsilon_{exp} + 11.82\ T_{exp} - 0.1116\ T_{stop}^2 + 17.85\ T_{stop} - 17,245$$

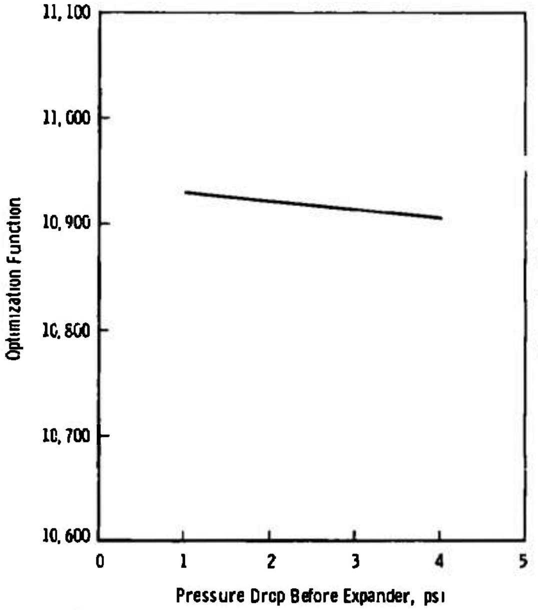

Figure 15. Effect of varying pressure drop before expander ($\dot{m}$ = 0.35).

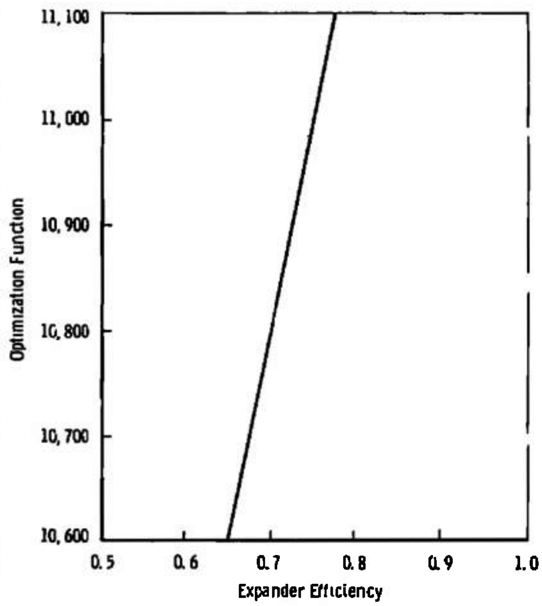

Figure 16. Effect of varying expander efficiency ($\dot{m}$ = 0.35).

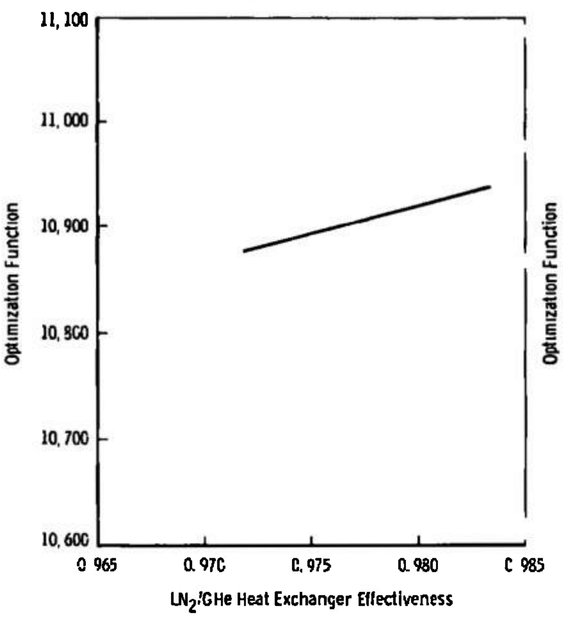

Figure 17. Effect of varying LN_2/GHe exchanger effectiveness ($\dot{m}$ = 0.35).

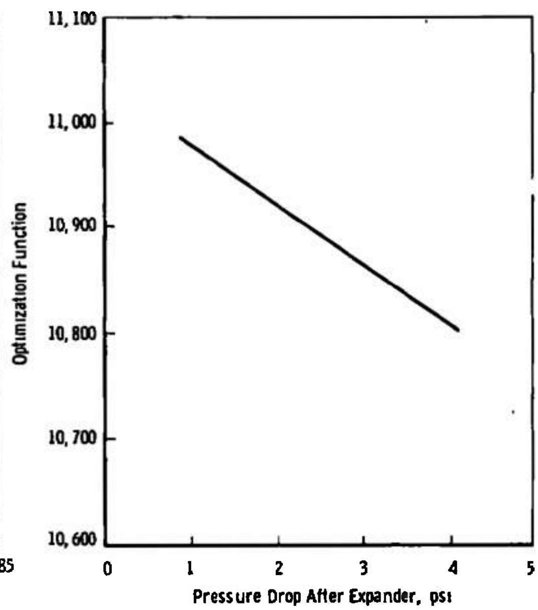

Figure 18. Effect of varying pressure drop after expander ($\dot{m}$ = 0.35).

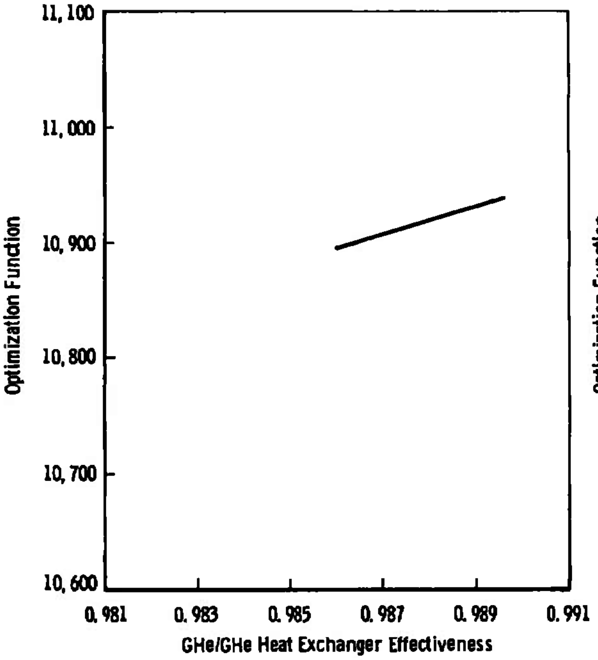

Figure 19. Effect of varying GHe/GHe heat exchanger effectiveness ($\dot{m}$ = 0.35)

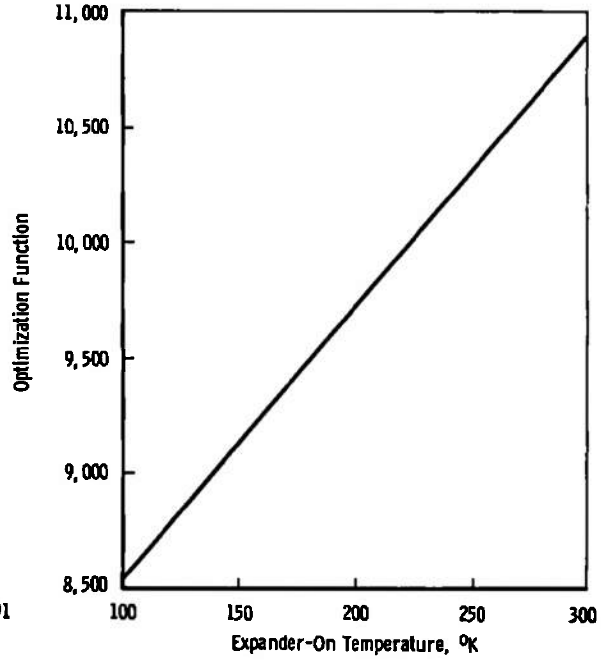

Figure 20. Effect of varying expander-on temperature ($\dot{m}$ = 0.35).

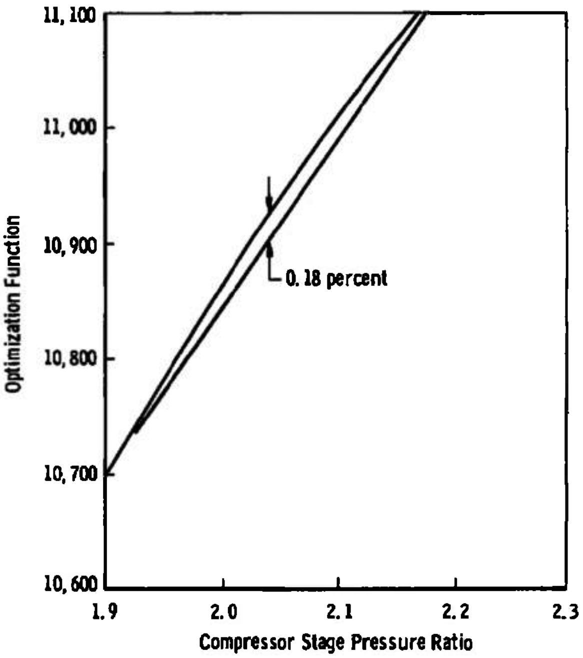

Figure 21. Effect of varying compressor stage pressure ratio ($\dot{m}$ = 0.35).

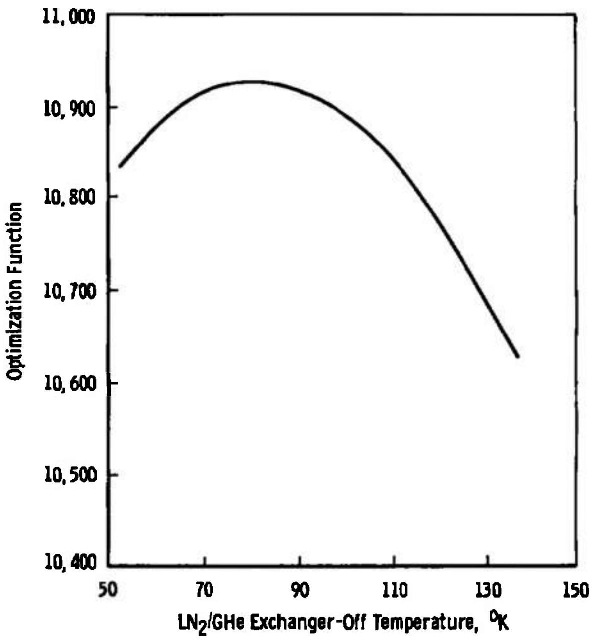

Figure 22. Effect of varying T_{stop} ($\dot{m}$ = 0.35).

By comparing the sensitivity terms (see Table 2) for the two different mass flow distributions, one can see that the stage pressure ratio is 6.4 times as sensitive with $\dot{m} = 0.35$ as it is with $\dot{m} = 1.565/\sqrt{T_9}$. However, changes in ϵ_{LN_2} are approximately 1.3 times less sensitive. With this modification, cooldown time could be reduced 24 percent.

Table 2. Mass Flow Distributions

Sensitivity Terms	$\dot{m} = 1.565/\sqrt{T_9}$	$\dot{m} = 0.35$
$\partial OPF/\partial P2P1$	232.4	1488
$\partial OPF/\partial \Delta P_{before\ expander}$	-1.279	-8.141
$\partial OPF/\partial \Delta P_{after\ expander}$	-9.017	-58.55
$\partial OPF/\partial \epsilon_{GHe}$	17,770	13,180
$\partial OPF/\partial \epsilon_{LN_2}$	6,492	4,949
$\partial OPF/\partial \epsilon_{exp}$	649	4,157
$\partial OPF/\partial T_{exp}$	$-0.072\ T_{exp}$	11.82
$\partial OPF/\partial T_{stop}$	$-0.069\ T_{stop} + 11.02$	$-0.223\ T_{stop} + 17.85$
$\partial OPF/\partial T_{LN_2}$	-68	(Not considered)

4.3 ADDING AN LN_2/GHe HEAT EXCHANGER

Adding a second LN_2/GHe heat exchanger in the middle of the existing GHe/GHe heat exchanger would be a relatively simple task. In the analytical model previously described, the GHe/GHe heat exchanger is a single unit. However, on the refrigerator that was tested there are two GHe/GHe heat exchangers in series. Transient performance calculations indicated that such a modification would reduce cooldown times by only 3.3 percent.

4.4 THE REHEAT CYCLE

A reheat cycle involves using two expanders and dividing the load into two sections. The cold gas from the first expander is routed through the first section of the load and back to the second expander. Cold gas from the second expander is routed through the remaining part of the

load. In the transient analysis it was assumed that full compressor mass flow would pass through both expanders and that the temperature of the gas entering both expanders was the same.

This modification would cut the cooldown time in half.

5.0 CONCLUSIONS

The conclusions of this study are as follows:

1. The inability of the expander in the GHe refrigerator of the Mark I Chamber to pass full compressor mass flow is the system's greatest deterrent to rapid cooldown.
2. An additional expander will allow full compressor mass flow to be utilized throughout the cooldown process and should reduce cooldown times by 24 percent.
3. Precooling the system's GHe/GHe heat exchanger prior to test can greatly reduce cooldown time.
4. Operator attention should be given to maintaining full mass flow through the load, particularly when the expander is being used. A more reliable gauge should be installed to allow this to be accomplished.
5. With the existing system the expander should be utilized when the gas temperature returning from the load reaches 115°K.
6. With the existing system the LN_2/GHe heat exchanger should be utilized until the gas temperature returning from the load reaches 80°K.
7. If future installations require larger load masses, cooldown times may become excessive. A reheat cycle using two expanders and existing equipment should reduce cooldown times to half their present value.

BIBLIOGRAPHY

Barron, Randall. Cryogenic Systems. McGraw-Hill Book Company, New York, 1966.

Gue, Ronald L. and Thomas, Michael E. Mathematical Methods in Operations Research. The Macmillan Company, New York, 1971.

Kropschot, R. H., Birmingham, B. W., and Mann, D. B., editors. Technology of Liquid Helium. U. S. Government Printing Office, Washington, D. C., 1968.

London, A. L., Biancardi, F. R., and Mitchell, J. W. The Transient Response of Gas-Turbine-Plant Heat Exchangers - Regenerators, Intercoolers, Precoolers, and Ducting. Transactions of the A.S.M.E., Easton, Pa., 1959.

Muhlenhaupt, R. C. and Strobridge, T. R. An Analysis of the Brayton Cycle as a Cryogenic Refrigerator. U. S. Government Printing Office, Washington, D.C., 1968.

NOMENCLATURE

C	Specific heat
Cp	Specific heat at constant pressure
g	Gravitational constant
k	Specific heat ratio
$\dot{m}$	Mass flow rate
OPF	Optimization function, proportional to cooldown time
P	Pressure, psi
P2P1	Compressor stage pressure ratio
Q	Thermal energy, or heat transferred
R	Gas constant
T	Temperature, °K
T_{exp}	Temperature of the gas coming from the load (T_{12}) at which the expander is added to the cycle, °K
T_{stop}	Temperature of the gas coming from the load (T_{12}) at which the LN_2/GHe heat exchanger is removed from the cycle, °K
t	Time
W	Mass
ΔP_{7-8}	A representative pressure drop preceding the expander
ΔP_{11-12}	A representative pressure drop following the expander

ϵ_{exp}	Expander effectiveness
ϵ_{GHe}	Heat exchanger effectiveness, GHe/GHe heat exchanger
ϵ_{LN_2}	Heat exchanger effectiveness, LN_2/GHe heat exchanger